Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MM05

Unit Mechanics 5

Thursday 12 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 A particle moves, with simple harmonic motion, in a straight line between two points, A and B. During this motion, its maximum speed is $1.3\,\mathrm{m\,s^{-1}}$. When the particle is $0.2\,\mathrm{m}$ from the midpoint of AB, its speed is $1.2\,\mathrm{m\,s^{-1}}$.

Find the distance between the points A and B.

[6 marks]

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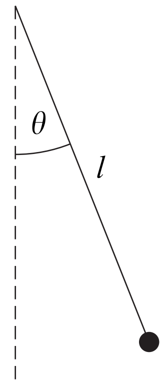
2 A simple pendulum consists of a small sphere of mass $m\,\text{kg}$ attached to one end of a light inextensible string of length l metres. The other end of the string is attached to a fixed point. The sphere is released from rest with the string taut and at an angle of $\frac{\pi}{10}$ to the downward vertical. When the pendulum has been in motion for t seconds, the angle between the string and the downward vertical is θ, as shown in the diagram.



(a) Show that the motion can be modelled by the differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\theta$$

and state clearly any assumptions that you make.

[4 marks]

(b) The length of the string is $39.2\,\text{cm}$.

(i) Find an expression for θ at time t.

[4 marks]

(ii) Find the time that it takes for θ to decrease from $\frac{\pi}{15}$ to $\frac{\pi}{30}$.

[4 marks]

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3 A particle moves under the action of a force which is always directed towards the origin, O, on a curve defined in plane polar coordinates as $r = \dfrac{3}{1 + \sin\theta}$.

(a) Find r when $\theta = 0$.

[1 mark]

(b) Given that the transverse component of the velocity of the particle is 5 when $\theta = 0$, show that $\dot{\theta} = \frac{5}{3}(1 + \sin\theta)^2$.

[4 marks]

(c) (i) Find $\dot{r}$ in terms of θ.

[2 marks]

(ii) Show that the radial component of acceleration is given by $\dfrac{k}{r^2}$, and state the value of the constant k.

[4 marks]

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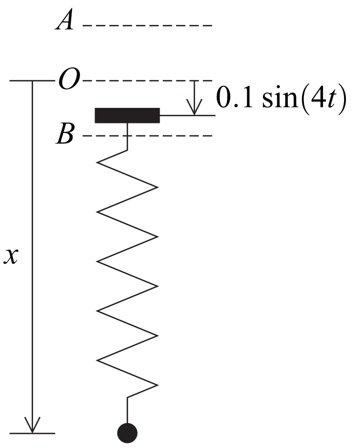
4 A spring has modulus of elasticity mg N and natural length 0.2 metres. A particle of mass m kg is attached to one end of the spring. The other end of the spring is attached to a peg which moves up and down between two points, A and B. The midpoint of AB is O. The point A is 0.1 metres above O, and B is 0.1 metres below O. At time t seconds, the displacement of the peg from the point O is $0.1\sin(4t)$ metres, where the downward direction is taken as positive. The displacement of the particle from O at time t is x metres, as shown in the diagram.



Assume that there is no air resistance.

(a) Show that

$$\frac{d^2x}{dt^2} + 49x = 19.6 + 4.9\sin(4t)$$

[5 marks]

(b) At time $t = 0$, the particle is at rest with $x = 0.4$. Find an expression for x at time t.

[10 marks]

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5 A rocket is launched from rest at the surface of the earth and rises vertically from rest. Assume that the only external force acting on the rocket is gravity. Assume that the acceleration due to gravity is constant at $g\,\mathrm{m\,s^{-2}}$.

As the rocket moves, fuel is ejected at a constant rate of $\lambda\,\mathrm{kg\,s^{-1}}$ and at a constant speed of $U\,\mathrm{m\,s^{-1}}$ relative to the rocket. The initial mass of the rocket, including fuel, is $M\,\mathrm{kg}$. At time t seconds, after the launch, the mass of the rocket and the remaining fuel is $m\,\mathrm{kg}$ and the velocity of the rocket is $v\,\mathrm{m\,s^{-1}}$.

(a) Show that, until the rocket runs out of fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{U}{m}\frac{\mathrm{d}m}{\mathrm{d}t} - g$$

[4 marks]

(b) Hence show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\lambda U}{M - \lambda t} - g$$

[2 marks]

(c) Express v in terms of g, M, t, U and λ.

[5 marks]

(d) Given that 80% of M is fuel, find in terms of g, M, U and λ the speed of the rocket when it runs out of fuel.

[4 marks]

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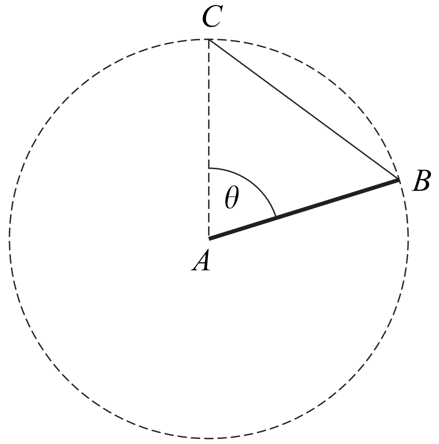
6 A rod, AB, of mass m and length a is pivoted at the end A and is free to rotate in a vertical plane. A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is attached to B and the other end to the point C, which is a distance of a directly above A. The angle between the rod and the upward vertical is θ, as shown in the diagram.



(a) The gravitational potential energy of the system, made up of the string and the rod, is taken to be zero at the level of A. Show that V, the total potential energy of the system, is given by

$$V = \frac{mga}{2}\left(12 - 7\cos\theta - 16\sin\left(\frac{\theta}{2}\right)\right) \quad \text{where } \frac{\pi}{3} \leqslant \theta \leqslant \frac{5\pi}{3}$$

[5 marks]

(b) Explain why the condition $\frac{\pi}{3} \leqslant \theta \leqslant \frac{5\pi}{3}$ has to be applied to the expression for V.

[1 mark]

(c) Find the values of θ for which the rod is in equilibrium.

[6 marks]

(d) Determine whether each of the values of θ found in part **(c)** corresponds to a position of stable or unstable equilibrium.

[4 marks]

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